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Influence of interlayer tunneling on the quantized Hall phases in intentionally disordered multilayer structures

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Abstract
Stability of the quantized Hall phases is studied in weakly coupled multilayers as a function of the interlayer correlations controlled by the interlayer tunneling and by the random variation of the well thicknesses. A strong enough interlayer disorder destroys the symmetry responsible for the quantization of the Hall conductivity, resulting in the breakdown of the quantum Hall effect. A clear difference between the dimensionalities of the metallic and insulating quantum Hall phases is demonstrated. The sharpness of the quantized Hall steps obtained in the coupled multilayers with different degrees of randomization was found consistent with the calculated interlayer tunneling energies. The observed width of the transition between the quantized Hall states in random multilayers is explained in terms of the local fluctuations of the electron density.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Considerable interest has emerged recently in the physics of the quantum Hall effect (QHE) in three-dimensional (3D) electron systems [1–3], in part motivated by the significant advances in the physics of graphene and few-layer graphite [4–6]. The existence of the energy gap is an essential condition for the quantization of the Hall conductors. Whenever the Fermi level is placed in the gap the QHE can be displayed [7]. This extends the QHE to 3D systems. The gaps in the energy spectra of quasi-particles arise due to a periodicity which may be a consequence of either atomic crystal structure or magnetic-field-induced density waves. Thus, the integer QHE was found and explained in periodic multilayer heterostructures [8] and in anisotropic 3D crystals [9–12]; in the latter case the formation of the density waves is highly favorable. Recently, the possibility of Hall conduction quantization was also indicated in isotropic 3D crystals [1]. In all of these cases the symmetry of the initial electron system was lowered to achieve the quantized Hall regime. A particularly interesting case is the artificial periodic multilayer systems where tailoring of the interlayer coupling allows examining theoretical predictions. The lowering of the host crystal symmetry by imposing the super-potential with a periodicity much larger than the lattice constant permits us to observe the QHE in these 3D structures in reasonably weak magnetic fields. Moreover, the interlayer coherence of the artificial multilayers may be managed by a random variation of the layer widths/compositions. In random multilayers the breaking of the symmetry responsible for the quantization of the Hall conductance destroys the QHE. In such a case the stability of the quantized Hall phases may be explored.

The central role of disorder in the quantization of the Hall conductivity was recognized soon after the discovery of the QHE [13]. The Anderson localization was found to be responsible for the appearance of the quantized plateaus of the Hall conductivity, while the transitions between the plateaus are determined by the extended states at the center of the Landau level (LL). However, whereas the disorder is vital for the QHE, it is also responsible for a breakdown of the QHE: a sufficiently strong disorder results in the disappearance of the quantized Hall phases. The disorder-induced destruction of the QHE in the two-dimensional electron gas (2DEG) has been extensively explored (see, for instance, [14, 15]). It is believed that the quantization of the Hall conductivity is destroyed when the disorder energy is large compared to the spacing between LLs [16]. Moreover, in order to explain a transition from an insulator at $B = 0$ (where all states are localized) to a quantum Hall conductor at finite B (where extended states exist below the Fermi level) the concept of the extended states flowing up due to disorder was introduced [17, 18]. The situation is somewhat different for 3D systems where at zero magnetic field, depending on the relation between the disorder energy (Δ) and the interlayer tunneling energy (t_z), the system may be in either an insulating or metallic state. In the latter case the LLs do not flow up, while they merge into a continuum with the decreasing magnetic field.

Furthermore, the successive magnetic-field-induced metal-to-insulator transitions which take place in the quantized Hall systems present the important case of the quantum phase transitions [19] where the metallic phase associated with the width of the interplateau range reveals the critical behavior [20, 21]. According to the theory [22], the interplateau width concerns a measure for the fraction of the extended states which contribute to the conduction at finite temperature. In 2DEG a single extended electron state results in a very sharp transition between the plateaus; the width of the transition vanishes as a power law with temperature [23, 24]. In an uncoupled periodic multilayer system each LL is additionally degenerated with the degeneracy equal to the number of layers (N). The interlayer coupling breaks this degeneracy, resulting in a finite width of the extended state even at zero temperature—Landau band (LB). Consequently, in coupled multilayers the interlayer tunneling plays an essential role in determination of the width of the plateau–plateau transitions and it can considerably influence the critical behavior of the metallic phase.

Besides the references cited above the magnetotransport properties of the multilayer electron systems in a strong magnetic field were also studied in [25–27] where the effect of the suppression of the interlayer charge transfer by in-plane field was found. However, as far as we know, in neither work were the influences of the interlayer tunneling and the interlayer correlations on the formation of the quantized Hall phases explored and that is the subject of this work.

This paper is composed as follows: samples, the experimental set-up and the method are all described in section 2. Theoretical considerations are presented in section 3. In section 4 the obtained results are analyzed and discussed. Conclusions are given in section 5.

2. Experimental details and method

In the multilayer samples reported here the width of the extended state was tuned by both the variation of the barrier thicknesses in periodic multilayers and by random variation of the wells. This allowed us to investigate the problem of the formation of the extended states in the quantized Hall regime and the conditions for the breakdown of the QHE in regimes not achievable in natural multilayer systems. It ought to be mentioned that recently the influence of the in-plane local

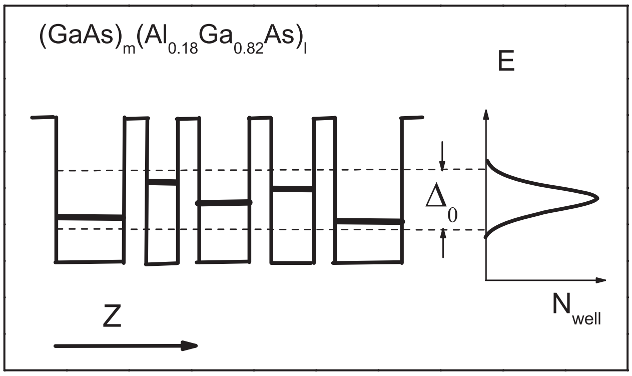

Figure 1. Structure of the intentionally disordered $(GaAs)_m(Al_{0.18}Ga_{0.82}As)_l$ multilayers.

variation of potential landscape on the transition between QHE states was studied in [28]. In our experiments the role of both the in-plane and the interlayer disorder on the width of the QHE transition was explored in a multilayer electron system; herewith the strength of the interlayer disorder was controlled during the sample growth.

The samples under investigation were $(GaAs)_m(AlGaAs)_l$ multilayer heterostructures. The multilayers were grown by molecular beam epitaxy at the same growth conditions on semi-insulating GaAs substrates. They consisted of GaAs undoped wells with thickness m monolayers (ML) and $Al_{0.18}Ga_{0.82}As$ barriers with thicknesses l ML. Two types of multilayer structures were studied: (i) uncoupled disordered multi-quantum wells (MQW) with barrier widths $l = 65$ and 100 MLs and (ii) weakly coupled disordered superlattices (SL) with barrier widths $l = 15$ MLs. The barriers of both the SLs and MQWs were n-doped with Si in the middle, leaving on both sides the undoped spacers with thickness 3 ML. Thus, the only difference between these SLs and MQWs was in the tunneling rate. In addition, in order to examine the role of electron mobility, the MQW structures were grown with spacers of 30 ML (high-mobility MQWs). In all cases the randomization was achieved by a random variation of the layer thickness m around the nominal value $m = 65$ ML according to a probability distribution which is obtained from a Gaussian probability density for the electron energy in the isolated well. The Gaussian is centered at the value of the electron energy corresponding to the 65 ML well and it is characterized by its full width at half-maximum Δ_0 (disorder energy). Unavoidable monolayer fluctuations cause $\Delta_0 = 0.23$ meV even in the nominally periodic $(GaAs)_{65}(Al_{0.18}Ga_{0.82}As)_l$ multilayer structures. The schematic structure of the random multilayers studied here is shown in figure 1.

The effect of the disorder in multilayer semiconductor structures strongly depends on screening. In weakly coupled random multilayers the screened random potential may be calculated as follows [29]:

$$\Delta = \frac{\Delta_0}{1 + 2\pi e^2 \rho_0 d/\epsilon} \tag{1}$$

where the unscreened (as-grown) disorder energy Δ_0 is reduced by the factor $\alpha = 1 + 2\pi e^2 \rho_0 d/\epsilon$, with ρ_0, d and ϵ

Table 1. Characteristic parameters of the random $(GaAs)_m(Al_{0.18}Ga_{0.82}As)_l$ multilayer structures measured at $T = 1.6$ K.

Sample	l (ML)	N	Δ_0 (meV)	E_F (meV)	n_{2D} ($\times 10^{11}$ cm^{-2})	μ (cm^2 V^{-1}s^{-1})
MQW1	100	20	0.23	24.8	8.4	7 500
MQW2	65	25	7.7	19.0	4.7	13 000
MQW3	65	25	7.7	19.0	4.7	12 000
MQW4	65	25	10.2	13.0	3.7	19 000
MQW5	65	25	14.8	11.0	3.2	9 500
MQW6[a]	100	20	0.23	22.6	6.6	72 000
MQW7[a]	100	20	2.6	22.0	4.7	54 000
MQW8[a]	100	20	7.7	22.0	6.6	40 000
MQW9[a]	100	20	14.8	21.5	6.5	75 000
SL1	15	25	0.23	10.1	3.3	7 000
SL2	15	25	0.95	9.6	3.2	5 700
SL3	15	25	2.6	9.4	3.1	5 100
SL4	15	25	5.2	8.0	2.7	6 500
SL5	15	25	7.7	8.5	2.9	5 900
SL6	15	25	9.4	8.3	2.6	6 600
SL7	15	25	10.2	7.5	2.3	3 370
SL8	15	25	14.8	6.7	2.2	4 100

[a] The multilayers with spacers 30 ML (high-mobility MQW).

being the density of states in a subband, the interlayer distance and the dielectric constant, respectively. The validity of this formula was quantitatively attested for disordered superlattices in [30]. The calculated screening factors α for the MQW and SL studied here are 8.2 (10.2 for the MQWs with the 100 ML barriers) and 5.5, respectively.

The transport measurements parallel to the layers were performed with a Perkin-Elmer 7280 lock-in amplifier at the temperature $T = 1.6$ K, frequency 1 Hz and current 10 μA in the samples patterned into Hall bars. The characteristic parameters of the studied structures are presented in table 1 where the Fermi energies were determined by the fits of the Shubnikov–de Haas oscillations, while the sheet densities were obtained from the QHE.

The magneto-resistance traces measured in one of the low-mobility uncoupled periodic multilayers (MQW1) are depicted in figures 2(a) and (b). At high magnetic fields the Hall resistivity measured in the periodic MQW develops clear plateaus. The plateau resistance at the filling factor ν can be calculated as $R_{xy} = h/e^2\nu N_{QHE}$. This expression assumes a parallel connection of N_{QHE} quantum wells in a multilayer structure. All 20 quantum wells contributed to the QHE in this MQW.

In order to quantitatively examine the influence of the interlayer tunneling on the transition between QHE states we determined the width of the transition between the plateaus ν and $\nu' = \nu + 2(\delta B_{\nu\nu'})$. We found that in the random multilayers a direct analysis of the plateau–plateau transition using the data of the Hall resistance is more appropriate than the method adopted for the 2DEG in [23] (where the derivative of the longitudinal magneto-resistance was used to extract the data of interest). Therefore, the derivatives of the Hall resistances ($\frac{dR_{xy}}{dB}$) were calculated and the peaks corresponding to the plateau–plateau transitional regions were associated with the corresponding interplateau widths. The dependences of these derivatives on the magnetic field are

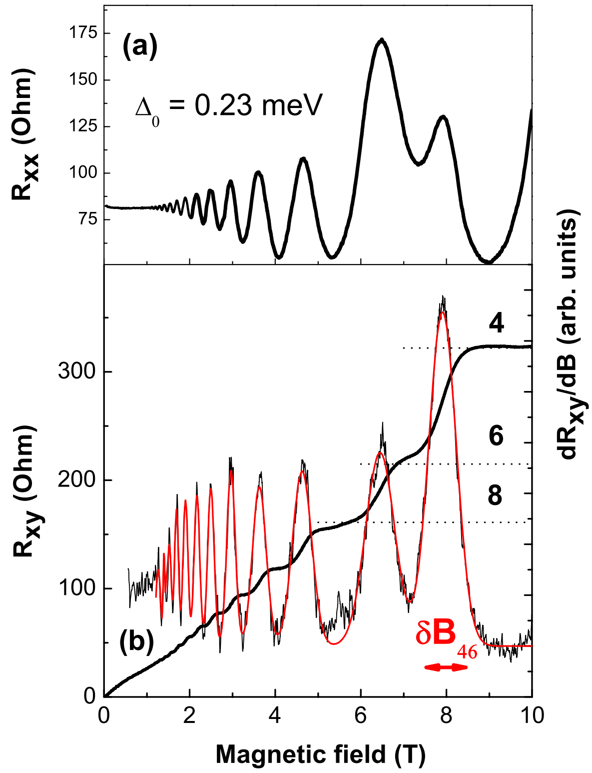

Figure 2. Longitudinal (a) and Hall (b) magneto-resistances measured at $T = 1.6$ K in the uncoupled periodic $(GaAs)_{65}(Al_{0.18}Ga_{0.82}As)_{100}$ multi-quantum well heterostructure (MQW). The numbers indicate the filling factors of corresponding plateaus. The derivative dR_{xy}/dB together with the best fit (dark gray (red online) line) are also shown in panel (b).

shown in figure 2(b). The interplateau widths were related to the corresponding widths of the Gaussian lines fitting the $\frac{dR_{xy}}{dB}$ dependences [31, 32]. The fit is shown by the gray line (red online) in figure 2(b). A correspondence between the widths measured in tesla and in eV is determined by the relation $\delta E(\text{eV}) = \frac{\hbar e}{m}(N - \frac{1}{2})\delta B_{\nu\nu'}(T)$, where $N = \nu'/2$ is the number of the corresponding Landau level. The interplateau broadening thus measured in the low-mobility MQW sets a reference for the corresponding broadenings measured in the weakly coupled random multilayers consisted of the same as in the MQW wells, where the width of the extended state is determined by the interlayer tunneling and/or by the interlayer disorder.

3. Theoretical description

The random potential produced by the disorder described previously provides a probe to measure the robustness of the quantized Hall state in coupled multilayer systems. On the one hand, the coherent interlayer coupling is responsible for a finite width LB in the periodic case; therefore one could expect the plateau–plateau transition to become sharper as the vertical disorder increases and so the tunneling contribution diminishes. However, on the other hand, different well thicknesses lead to different occupation of the quantum wells. For this reason a smoothing of the quantized Hall plateaus occurs and then a broadening of the interplateau distance, with the increasing interlayer disorder, must be expected. Thus in the random coupled multilayers both the interlayer tunneling and the interlayer disorder contribute to the interplateau broadening. The effect of the interlayer disorder on the transition between the plateaus ν and $\nu'(\delta B_{\nu\nu'})$ may be attributed to the interlayer fluctuations of sheet electron density (δn_{2D}) as $\delta B_{\nu\nu'} = \frac{h}{e\nu^*}\delta n_{2D}$, where $\nu^* = \nu' - 1$ corresponds to a critical filling factor of the quantized Hall conductor phase of random multilayers related to the middle of the corresponding half-filled LB.

The disorder-induced interlayer fluctuations of sheet electron density may be estimated as $\delta n_{2D} \simeq n_{2D}\frac{\Delta}{E_F}$. Thus, the interplateau broadening in energy, caused by these fluctuations, can be written as

$$\delta \simeq \frac{\pi\hbar^2}{m^*}\frac{n_{2D}}{E_F}\Delta. \quad (2)$$

In the SLs, the contribution of the coherent tunneling has been estimated via an interlayer tunneling energy (t_z) accounting directly for the energy broadening of the LB. The calculation of t_z has been carried out using the transfer matrix technique, modeling the multilayer structure by a one-dimensional Kronig–Penney potential. The relevant parameters used for the simulation are as follows: effective mass in barriers $m_b = 0.083m_0$, effective mass in wells $m_w = 0.067m_0$, and conduction band offset $V = 124$ meV and 3.0 Å for the monolayer width. For a given vertical disorder several different realizations of the system have been considered. Each sequence includes 25 periods with constant barriers of 15 ML and wells with different widths according to a probability distribution obtained from a Gaussian distribution for the electron energy in the isolated well. The Gaussian is centered at the value of the electron energy corresponding to the 65 ML well and its half-width is the disorder energy Δ. Since there is no straightforward method to estimate the coupling strength in terms of energy in the intentionally disordered multilayers, a reasonable estimation can be done by studying the electronic localization length as a function of the energy. One would expect the interlayer coupling energy t_z to be proportional to the energy interval W_E where the vertical localization length of the electrons is larger than a given threshold length Lth, which we chose to be 43.5 nm, corresponding to the average length of the composition well–barrier–well; that is, we assume that only states whose localization length covers a couple of wells can contribute to coherent tunneling. Nevertheless since the localization length within W_E is not constant (not all energies give the same contribution to tunneling), it seems reasonable to include a correction factor accounting for this dependence. The interlayer coupling energy for each sequence was estimated as $t_z = (\frac{\Lambda_{eff}}{L})W_E$, where the correction factor is the ratio between the effective localization length Λ_{eff} of the electrons within W_E and the total length of the system L. When the disorder goes to zero and the length of the system grows, the effective localization length tends to equal the total length of the system and the interval W_E will converge to the miniband width, which is the limiting value one would expect for the interlayer coupling energy for the infinite periodic structure; the interlayer tunneling energy of the periodic SL

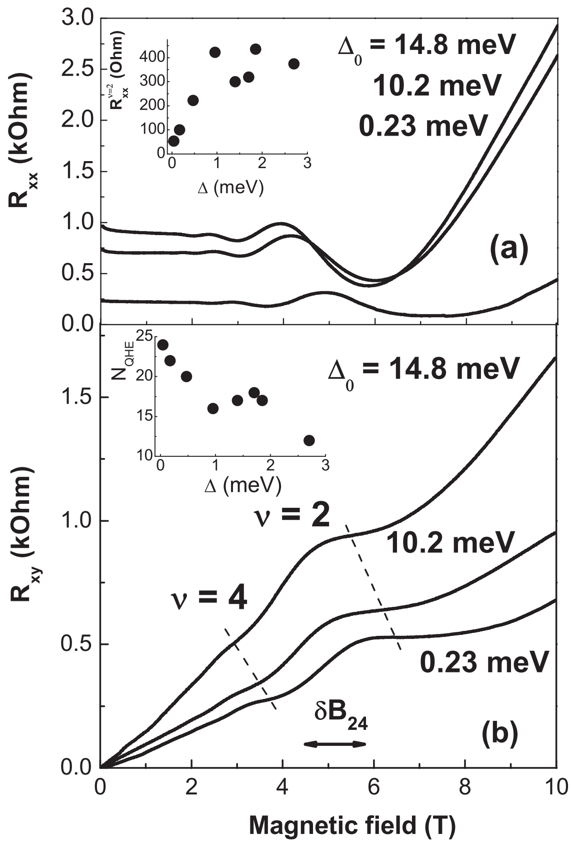

Figure 3. Longitudinal (a) and Hall (b) magneto-resistances measured at $T = 1.6$ K in the random weakly coupled $(GaAs)_m(Al_{0.18}Ga_{0.82}As)_{15}$ multilayer structures (SLs) with different disorder strengths. The insets in panels (a) and (b) show the longitudinal resistance R_{xx} at the minimum corresponding to $\nu = 2$ measured in differently disordered superlattices and the number of quantum wells contributing to the QHE, respectively.

was calculated to be $t_z = 1.5$ meV. The effective localization length Λ_{eff} for the disordered sequences is obtained from the transmission efficiency T_{eff} (the area enclosed by the transmission coefficient per energy unit) inside W_E, using the well-known Kirkman and Pendry formula: $\Lambda_{eff} = -2L/\log(T_{eff})$. The localization length calculated in this way was found in good agreement with the one experimentally determined in intentionally disordered SLs in [33].

Hence, the total interplateau width, and then the dependence of the interplateau distance as a function of disorder, can be estimated as a sum of the interlayer tunneling and the interlayer disorder contributions plus the broadening due to the in-layer scattering potential (Γ):

$$\delta E = t_z + \delta + \Gamma. \quad (3)$$

4. Results and discussion

The dependences of the longitudinal and Hall resistances on the magnetic field measured in some of the weakly coupled multilayers with different strengths of the interlayer disorder are shown in figure 3. The quantized plateaus were found at the filling factors $\nu = 2$ and 4. Contrary to the MQW the

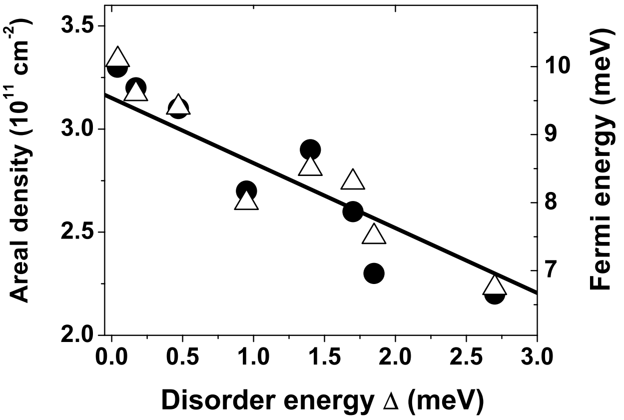

Figure 4. Dependence of the areal densities obtained by the $\nu = 2$ (closed circles) plateaus and of the Fermi energies (open triangles) measured by the Shubnikov–de Haas oscillations measured in weakly coupled multilayers (SLs) as functions of the disorder energy; the line was calculated as explained in the text.

high-index plateaus were not observed in the SLs because no gaps were opened in the electron energy spectrum at weak magnetic field ($\hbar\omega_c \lesssim t_z$). The following consequences of the intentional disorder can be distinguished: with the increasing disorder (i) the minimum longitudinal resistances corresponding to the quantized Hall phase at $\nu = 2$ enhance (shown in the inset to figure 3(a)) and (ii) more wells become empty (the decreasing N_{QHE} is shown in the inset to figure 3(b)) and the electron concentration decreases; this is due to the redistribution of the electrons over the random potential (some of the wells become empty while the Fermi energy decreases with increasing disorder). While the fluctuations of the local (interlayer) electron density cause both the broadening of the interplateau transition and the increase of minimum longitudinal resistance. The quantized Hall plateau at $\nu = 4$ disappears in the most disordered sample. In that case the disorder energy $\Delta \simeq 0.5\hbar\omega_c$ which confirms the Thouless criterion for breakdown of the QHE [16].

The dependences of the areal electron density on the disorder energy obtained from the $\nu = 2$ plateaus in SLs with different disorder strengths are shown in figure 4. The observed decreasing areal density is caused by the redistribution of the electrons over the random potential (already mentioned above) which leads to a lowering of the Fermi energy $E_F = E_{F0} - \Delta$, where E_{F0} is the Fermi energy in an unperturbed sample [34]. Consequently, the areal density is $n_{2D} = n_0(1 - \Delta/E_{F0})$. A good fit to the experimental results was obtained with the value of the Fermi energy $E_{F0} = 9.6$ meV. As expected for the two-dimensional electrons, the variation of the areal density well corresponds to the change of the Fermi energy determined by the low-field magneto-resistance oscillations. The ratio n_{2D}/E_F determined by these data was used in the calculations of the broadening energy δ. Hence, the insulating phase of the weakly coupled multilayers shows a two-dimensional behavior.

Additionally to the different number of plateaus found in the MQWs and the SLs, their different behaviors are also manifested by the different temperature dependences

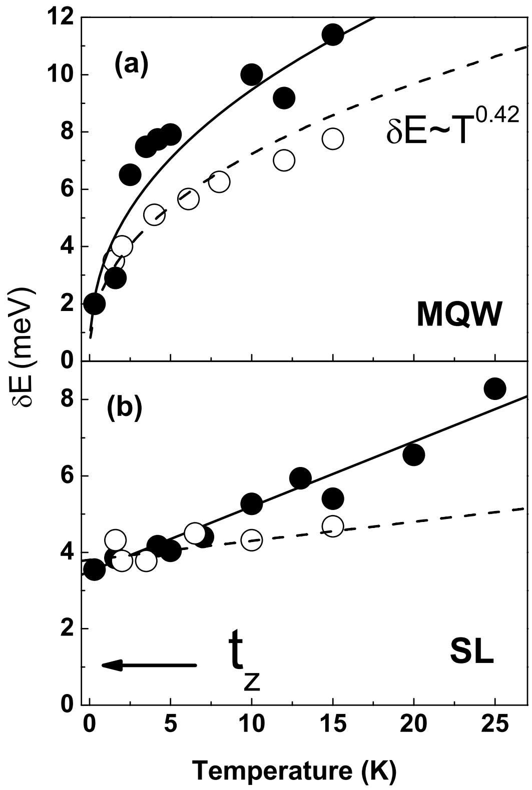

Figure 5. Interplateau widths measured at different temperatures (a) in the periodic (MQW1) and random (MQW5) isolated multilayers and (b) in the periodic (SL1) and random (SL8) weakly coupled multilayers. The data obtained in the periodic and random structures are shown by closed and open circles, respectively. The lines in (a) were calculated as explained in the text while the lines in (b) are guides for the eyes.

of the interplateau broadenings depicted in figures 5((a) and (b)). According to the scaling theory [22] the interplateau broadening changes with temperature as T^a with the exponent $a = p/2\mu$, where p and μ are the exponents in the temperature dependences of the elastic time and of the localization length, respectively. This temperature divergence points to a single energy of the extended state corresponding to the LL. Indeed, as shown in figure 5(a) the interplateau broadening of the MQW reveals the predicted temperature dependence with $a = 0.42$ as in [23]. This shows the two-dimensional character of the metallic system formed in the quantized Hall phase of the MQW. At the same time the width of the interplateau broadening of the periodic SL depicted in figure 5(b) behaves as a linear function of the temperature. The value of the interplateau width extrapolated to zero temperature indicates the tunneling energy plus the in-layer scattering potential Γ. The localization length exponent was calculated in weakly coupled multilayers in [35–39] as $\mu = 1.4$–1.7. Using the average localization length exponent $\mu = 1.55$ together with the experimental value $a = 1$ determined in the SL, we found the elastic time exponent $p \approx 3$ as due to the electron–electron interaction in the bulk, which is in good agreement with the value obtained by the weak localization measurements in similar GaAs/AlGaAs SLs in [40]. Therefore, contrary to the 2DEG the metallic quantized Hall phase of the coupled multilayers reveals a three-dimensional character. As shown in figure 5(a) and (b), where open circles correspond to the data obtained in the random MQW and SL, respectively, the dimensionality of the metallic phase does not change with the disorder.

It is worth adding that the 3D–2D transition induced by an in-plane magnetic field was observed in [25–27]. In this case the dimensionality of the electron system was lowered because the in-plane magnetic field decreased the overlap of the electron wavefunctions of adjacent quantum wells, consequently reducing the interlayer coupling. In contrast, we demonstrated the alteration of the dimensionality of the quantized Hall phases which takes place with the change of the magnetic field perpendicular to the layers.

Thus we can state that the influence of the interlayer tunneling on formation of the quantized Hall states is manifested by the following: (i) the number of the plateaus observed in MQWs and in SLs is different (no more than two plateaus were observed in SLs, while much more of them were found in MQWs) and (ii) different temperature dependences of the transitional energies, consistent with the theory, were observed in MQWs and SLs.

The interplateau widths obtained in SLs are demonstrated in figure 6(a) as a function of the effective screened random potential. In qualitative agreement with the theory, the interplateau width decreases with the increasing interlayer disorder following the calculated tendency of the tunneling rate. Then, after a certain disorder strength the interplateau transition begins to broaden and the interlayer disorder dominates. The interplateau widths obtained in MQW structures are also shown in figure 6(b). The mobility does not seem to affect the interplateau widths measured in the MQW structures (the data shown by closed and open triangles were obtained in low- and high-mobility MQWs, respectively). This indicates the dominant contribution of the short-range scattering to the interplateau width measured in the MQWs.

The tunneling energy (t_z) and the broadening energy ($\delta + \Gamma$) calculated as functions of the disorder energy are shown in figure 6(a) by the vertical black bars and by the solid line, respectively. The total estimated dependence of the interplateau width (δE) is shown by the vertical gray (red online) bars. When calculating the total interplateau broadening the contribution of the broadening due to the in-layer scattering potential ($\Gamma = 0.8$ meV) independent of the interlayer disorder was added to obtain the best fitting to the experimental data. On the other hand, the interplateau broadening of the random isolated MQW, where $t_z = 0$, is determined by the broadening energy $\delta + \Gamma$. The data shown in figure 6(b) reveal the same as in the disordered SLs tendency of increasing interplateau broadening with the disorder energy Δ predicted by equation (2).

Thus, we affirm good agreement obtained in both the SLs and the MQWs in the range of strong interlayer disorder ($\Delta >$ 1 meV). At smaller disorder the experimental data reveal an extra contribution. It is possible that the enhanced width of the interplateau transition observed in weakly disordered SLs and MQWs may be caused by the disorder-induced spin-assisted interlayer tunneling shown to result in the intermediate metallic

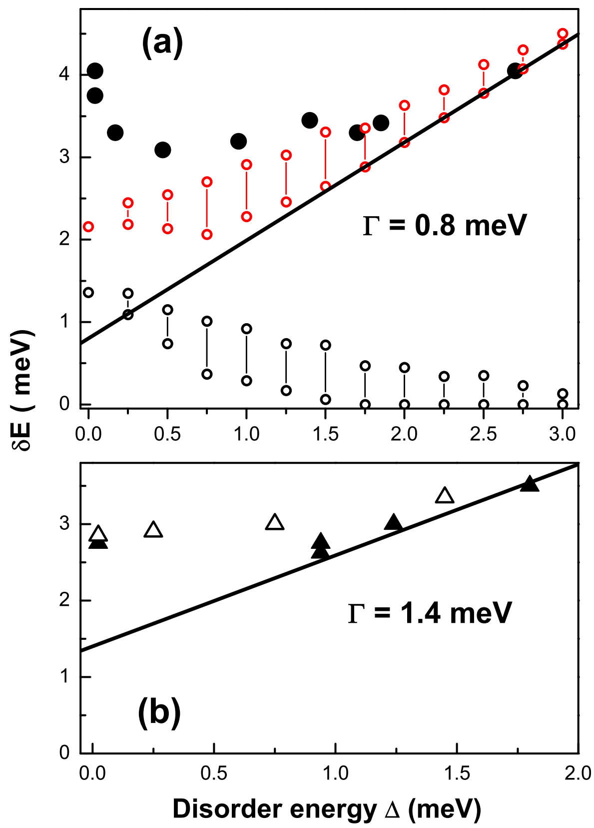

Figure 6. Interplateau widths δE obtained in the random $(GaAs)_m(Al_{0.18}Ga_{0.82}As)_{15}$ multilayer structures with different disorder strengths: (a) SLs for $\nu = 2$ (closed circles) and (b) low- and high-mobility MQWs for $\nu = 4$ (open and closed triangles, respectively). The vertical black bars mark the intervals including the calculated interlayer coupling energies t_z for the SLs, considering 100 different realizations of the system for each value of the disorder, while the solid lines demonstrate the disorder-induced interplateau broadening δ calculated in SLs and MQWs. The total dependence of the interplateau width on disorder energy calculated according to equation (3) is shown by the dark gray (red online) vertical bars (b).

phase separating the metallic and insulating quantized Hall phases [35]. According to the theory, such a metallic phase occurs even for an infinitesimal interlayer tunneling resulting in broader transitions between quantized Hall plateaus even at zero temperature. The interlayer disorder destroys the interlayer coherence causing the observed agreement between the experimental and calculated data.

It ought to be mentioned that the question of the critical behavior of the quantized Hall system is of fundamental importance to understand the quantum phase transitions [41, 19]. However, the experimental data published so far reveal different critical exponents a which questions the universality of the theory (see the discussion in [19] and references therein). In fact, as was recently demonstrated [19], the variety of critical exponents observed experimentally may relate to different percolation processes involving dephasing and quantum tunneling between the quantum Hall droplets. The phase diagram thus developed explains the different exponents observed. We did not find a considerable effect of the mobility on the critical exponent a, similar to that observed in [24]. Moreover, our data do not reveal any significant change of the critical exponent with the variation of the interlayer disorder. This implies a universal scaling relation for the magneto-resistance and conductance ruled by the quantum percolation predicted in [22] and observed in [23].

5. Conclusion

The effects of the interlayer tunneling and disorder on the quantum Hall effect were studied in multilayers. The tunneling was found to affect the temperature dependences of the interplateau transitional energies which were observed to be different in the isolated and coupled multilayers. This implies a different critical behavior of the corresponding metallic phases. In the coupled multilayers the quantized Hall phases reveal different dimensionalities: the insulating phase is two-dimensional, as in the 2DEG, while the metallic phase is three-dimensional. The observed modification of the quantized Hall phases agrees with the phase diagram of the 3D quantized Hall conductors developed in [38, 35]. The important role of the electron density fluctuations in determination of the transition between the quantized Hall phases was established. The possible signature of the spin-assisted interlayer tunneling predicted in multilayers in [35] is indicated.

Acknowledgments

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